Indefinite Integrals

Integration may be regarded as the inverse Process of Differentiation

The word integration literally means Summation and this is why the Symbol of integration $\int$ being regarded as the initial letter of the word Sum.

Primitive Function - A Function $F(x)$ is called Primitive function of $f(x)$ if $F'(x) = f(x)$

thus we may say the Process of Finding Primitive Function or the integral of any given differential is called integration

Ex:→ $\frac{d}{dx}(\sin x) = \cos x$

So the primitive Function of $\cos x$ is the Function of $\sin x$.

thus To find the integral of $f(x)$, we have just to find out the function whose differential is $f(x)$

Further

As $f(x)$ and $f(x)+C$ have the Same differential co-efficient say $g(x)$,

So we may write

$$\int g(x)\,dx = f(x) + C$$

where C is called the constant of integration

By taking different values of c we can get any No of solutions from $f(x)+c$. This is why we call $\int g(x)dx$ an indefinite integral

Methods of integration :→

Since all the integrands are not always in standard forms. When the integrand is not in standard form, we change it in standard forms by the use of one or more of the following methods

(i) Method of Transformation

(ii) Method of substitution

(iii) Method of integration by parts.

Method of Transformation

The given integrals can be reduced to standard forms by suitable transformation or by using Trigonometric formula

Algebraic Transformation —

Standard integral —

(i) $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}$ when $n+1 \neq 0$

(ii) $\int x^n dx = \frac{x^{n+1}}{n+1}$

(iii) $\int \frac{1}{ax+b}\,dx = \frac{1}{a}\log|ax+b|$ (v) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x$

(iv) $\int \frac{1}{x}\,dx = \log|x|$ (vi) $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x$

(vii) $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x$

Algebraic Transformation –

(i) $$\int \frac{dx}{\sqrt{ax+b} \pm \sqrt{ax+c}} = \int \frac{\sqrt{ax+b} \mp \sqrt{ax+c}}{b-c}\,dx$$

NOTE – co-efficient of x under sign of surds should be same

$$= \frac{1}{b-c}\left[\int (ax+b)^{\frac{1}{2}}\,dx \mp \int (ax+c)^{\frac{1}{2}}\,dx\right]$$

... which are in standard form.

(ii) $\int f(x)\,[g(x)]^n\,dx$ When $g(x)$ are in standard form.

for say, $f(x) = A\,g(x) + B$. —— (1)

Then $f(x)\,[g(x)]^n = [A\,g(x) + B]\,[g(x)]^n$

$= A\,[g(x)]^{n+1} + B\,[g(x)]^n$

$\therefore \int f(x)\,[g(x)]^n\,dx = A\int [g(x)]^{n+1}\,dx + B\int [g(x)]^n\,dx$

for values of A and B put $x=0$ and $x=1$ in (1) successively and get simultaneous equation in A and B

On solving we get values of A and B

Trigonometric Transformation —

Standard integral :-

$\int \sin kx \, dx = -\frac{1}{k} \cos kx$ $\quad \int \cos kx \, dx = \frac{1}{k} \sin kx$

$\int \sec^2 kx \, dx = \frac{1}{k} \tan kx$ $\quad \int \operatorname{cosec}^2 kx \, dx = -\frac{1}{k} \cot kx$

$\int \sec kx \tan kx \, dx = \frac{1}{k} \sec kx$ $\quad \int \operatorname{cosec} kx \cot kx \, dx = -\frac{1}{k} \operatorname{cosec} kx$

Trigonometric Transformation.

$\sin^2 kx = \frac{1}{2} [1 - \cos 2kx]$ $\quad \cos^2 kx = \frac{1}{2} \{1 + \cos 2kx\}$

$\tan^2 kx = [\sec^2 kx - 1]$ $\quad \cot^2 kx = \operatorname{cosec}^2 kx - 1$

$\sin kx \cos kx = \frac{1}{2} \sin 2kx$

$\sin^3 kx = \frac{1}{4} [3 \sin kx - \sin 3kx]$ $\quad \cos^3 kx = \frac{1}{4} [3 \cos kx + \cos 3kx]$

$\sqrt{1 + \cos kx} = \sqrt{2} \cos \frac{kx}{2}$ $\quad \sqrt{1 + \sin kx} = \cos \frac{kx}{2} + \sin \frac{kx}{2}$

$\sqrt{1 - \cos kx} = \sqrt{2} \sin \frac{kx}{2}$ $\quad \sqrt{1 - \sin kx} = \cos \frac{kx}{2} - \sin \frac{kx}{2}$

$$\frac{1 + \cos kx}{1 - \cos kx} = \cot^2 \frac{kx}{2} = \operatorname{cosec}^2 \frac{kx}{2} - 1$$

$$\frac{1 - \cos kx}{1 + \cos kx} = \tan^2 \frac{kx}{2} = \sec^2 \frac{kx}{2} - 1$$

Transformation of Expression of cos and Sine in the T. ratio of Compound angle.

$$a \cos\theta \pm b \sin\theta = \sqrt{a^2+b^2} \left[\frac{a}{\sqrt{a^2+b^2}} \cos\theta \pm \frac{b}{\sqrt{a^2+b^2}} \sin\theta \right]$$

use of the formulae.

$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$ $\quad \sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$

$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$ $\quad \cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$

using Above we Transform the T- integral in Standard T- integral

Exponential Transformation

Standard Integral - (i) $\int e^{kx}\,dx = \frac{1}{k}e^{kx}$

(ii) $\int a^{x}\,dx = \int e^{x\log a}\,dx$

$= \frac{1}{\log a} e^{x\log a} = \frac{1}{\log a}\cdot a^{x}$

(iii) $\int e^{\lambda \log x} = x^{\lambda}$.

(iv) (a) $\int \frac{e^{kx}+1}{e^{kx}-1}\,dx = \int \frac{e^{\frac{kx}{2}}+e^{-\frac{kx}{2}}}{e^{\frac{kx}{2}}-e^{-\frac{kx}{2}}}\,dx$

(b) $\int \frac{e^{kx}-1}{e^{kx}+1}\,dx = \int \frac{e^{\frac{kx}{2}}-e^{-\frac{kx}{2}}}{e^{\frac{kx}{2}}+e^{-\frac{kx}{2}}}\,dx$

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By - Samrendra kumar.